GAUGE THEORIES OF GRAVITATION
A Reader with Commentaries

Editors

Milutin Blagojević
Institute of Physics, University of Belgrade

Friedrich W. Hehl
Institute of Theoretical Physics, University of Cologne, and
Department of Physics and Astronomy, University of Missouri, Columbia

Foreword by
T. W. B. Kibble, FRS

Imperial College Press, London, April 2013
Classification of gauge theories of gravity

\[ \text{Part C} \]

\[ \text{MAG} \]

\[ \varnothing \neq 0 \]

\[ \text{WG} \]

\[ \text{tr} \ Q \neq 0 \]

\[ \text{CG} \] \quad \text{AdSG} \quad \text{SuGra} \]

\[ \text{Part B} \]

\[ \text{R} = 0 \]

\[ \text{TG} \]

\[ \text{EC} \]

\[ \text{GR}_{\parallel} \quad \text{GR} \]

\[ \text{PG} \]

\[ \text{GR} = \text{general relativity (Einstein's theory of gravity)}, \]
\[ \text{TG} = \text{translation gauge theory (of gravity) aka teleparallel theory (of gravity)}, \]
\[ \text{GR}_{\parallel} = \text{a specific TG known as teleparallel equivalent of GR (spoken “GR teleparallel”)}, \]
\[ \text{WG} = \text{Weyl(-Cartan) gauge theory (of gravity)}, \]
\[ \text{MAG} = \text{metric-affine gauge theory (of gravity)}, \]
\[ \text{CG} = \text{conformal gauge theory (of gravity)}, \]
\[ \text{AdSG} = \text{(anti-)de Sitter gauge theory (of gravity)}, \]
\[ \text{SuGra} = \text{supergravity (super-Poincaré gauge theory of gravity)}. \]

\[ \text{PG} = \text{Poincaré gauge theory (of gravity)}, \]
\[ \text{EC} = \text{Einstein–Cartan (–Sciama–Kibble) theory (of gravity)}, \]

The symbols in the figure have the following meaning: rectangle □ → class of theories; circle ◯ → definite viable theories; nonmetricity \( Q = \varnothing + \frac{1}{4}(\text{tr} \ Q)1 \), torsion \( T \), curvature \( R \).
Contents

Foreword by T. W. B. Kibble  ix
Preface  xi
Acknowledgments  xiii
List of Useful Books  xv

Part A  The Rise of Gauge Theory of Gravity up to 1961  1

1. From Special to General Relativity Theory  3
Commentary
1.1 A. Einstein, The foundation of the general theory of relativity,  Annalen der Physik 49, 769–822 (1916); extract

2. Analyzing General Relativity Theory  17
Commentary
2.1 E. Cartan, On a generalization of the notion of Riemann curvature and spaces with torsion (in French),  Comptes Rendus Acad. Sci. (Paris) 174, 593–595 (1922)
2.2 E. Cartan, Space with a Euclidean connection, in: E. Cartan,  Riemannian Geometry in an Orthogonal Frame, Lectures given at the Sorbonne 1926–27 (World Scientific, River Edge, NJ, 2001); extract
2.4 E. Stueckelberg, A possible new type of spin-spin interaction,  Phys. Rev. 73, 808–808 (1948)
2.5 H. Weyl, A remark on the coupling of gravitation and electron,  Phys. Rev. 77, 699–701 (1950)

3. A Fresh Start by Yang–Mills and Utiyama  71
Commentary


Part B Poincaré Gauge Theory

4. Einstein–Cartan(–Sciama–Kibble) Theory as Viable Gravitational Theory

Commentary


5. General Structure of Poincaré Gauge Theory (Including Quadratic Lagrangians)

Commentary


6. Translational Gauge Theory

Commentary

6.1 G. D. Kerlick, Spin and torsion in general relativity: foundations, and implications for astrophysics and cosmology, Ph.D. Thesis (Princeton University, Princeton, NJ, 1975); extract

6.3 Y. Itin, Energy-momentum current for coframe gravity, *Class. Quantum Grav.* 19, 173–190 (2002); extract

7. Fallacies About Torsion 259
   Commentary

**Part C Extending the Gauge Group of Gravity** 281

   Commentary

9. From the Poincaré to the Affine Group: Metric-Affine Gravity 311
   Commentary

10. Conformal Gauge Theory of Gravity* 361
    Commentary
11. (Anti-)de Sitter Gauge Theory of Gravity\textsuperscript{*}  
Commentary  

12. From the Square Root of Translations to the Super Poincaré Group  
Commentary  

Part D Specific Subjects of Metric-Affine Gravity and Poincaré Gauge Theory  

13. Hamiltonian Structure  
Commentary  

14. Equations of Motion for Matter\textsuperscript{*}  
Commentary  
15. Cosmological Models

Commentary


16. Exact Solutions

Commentary


17. Poincaré Gauge Theory in Three Dimensions

Commentary

17.2 M. Blagojević and B. Ćvetković, Black hole entropy in 3D gravity with torsion, Class. Quantum Grav. 23, 4781–4795 (2006)

18. Dislocations and Torsion

Commentary

18.2 R. A. Puntigam and H. H. Soleng, Volterra distortions, spinning strings, and cosmic defects, Class. Quantum Grav. 14, 1129–1149 (1997); extract

19. The Yang Episode: A Historical Case Study

Commentary

19.2 Chen Ning Yang, Selected Papers 1945–1980, with commentary (World Scientific, Singapore, 2005); extract

19.4 J.-P. Hsu and D. Fine (eds.), *100 Years of Gravity and Accelerated Frames, The Deepest Insights of Einstein and Yang–Mills* (World Scientific, Hackensack, NJ, 2005); extract

Symmetry has always played a big role in physics. Advancing understanding has time and again revealed previously unknown symmetries. Isaac Newton abandoned the idea of a preferred origin of space, revealing the underlying translational symmetry; Albert Einstein uncovered an unexpected symmetry between time and space.

A key innovation of the twentieth century was Hermann Weyl’s invention of gauge theory, in which a global physical symmetry is replaced by a local one; the arbitrary phase in the quantum wave-function becomes a function of space and time, a change that requires the existence of the electromagnetic field. This proved to be an astonishingly fruitful idea. Today, all the components of the “standard model” of particle physics that so accurately describes our observations are gauge theories. Weyl’s “gauge principle”, that global symmetries should be promoted to local ones, applied to the standard-model symmetry group $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$, is enough to yield the strong, weak and electromagnetic interactions.

Only gravity is missing from this model. But it too shows many of the same features. Going from special to general relativity involves replacing the rigid symmetries of the Poincaré group—translations and Lorentz transformations—by freer, spacetime dependent symmetries. So it was natural to ask whether gravity too could not be described as a gauge theory. Is it possible that starting from a theory with rigid symmetries and applying the gauge principle, we can recover the gravitational field? The answer turned out to be yes, though in a subtly different way and with an intriguing twist. Starting from special relativity and applying the gauge principle to its Poincaré-group symmetries leads most directly not precisely to Einstein’s general relativity, but to a variant, originally proposed by Élie Cartan, which instead of a pure Riemannian spacetime uses a spacetime with torsion. In general relativity, curvature is sourced by energy and momentum. In the Poincaré gauge theory, in its basic version, there is also torsion, sourced by spin.

As someone who was involved in the early stages of this development, I am astonished and intrigued by how the theory has developed over the last half century. Reading this book makes it clear how wide its ramifications have spread. Over the years, Poincaré gauge theory has been put on a much firmer mathematical base. In its simplest form, it gives predictions that are in almost all observational situations identical with those of general relativity, but in situations of extremely high density there are significant differences. These differences
may be of profound importance for the physics of the very early universe and of black holes, and could one day be subject to observational test.

Moreover, Poincaré gauge theory is not necessarily the end of the story. There are several possible extensions, in which the basic symmetry group is even larger; the Poincaré group may be augmented by the inclusion of dilatations or even enlarged to the full group of affine transformations. The resulting theories, the Weyl-Cartan theory and the metric-affine gravity theory, have some very attractive features. Only time will tell whether any of these intriguing theories is correct and which of the hypothesized hidden symmetries is actually realized in nature. For anyone interested in pursuing these ideas, this book certainly provides a fascinating and very valuable resource.
Preface

We have been both fascinated by gauge theories of gravity since the 1960s and the 1970s and have followed the subject closely through our own work. In this reprint volume with commentaries we would like to pass over our experience to the next generation of physicists. We have tried to collect the established results and thus hope to prevent double work and to focus new investigations on the real loopholes of the theory.

The aim of this reprint volume with commentaries is to introduce graduate students of theoretical physics, mathematical physics or applied mathematics, or any other interested researcher, to the field of classical gauge theories of gravity. We assume that our readers are familiar with the basic aspects of classical mechanics, classical electrodynamics, special relativity (SR), and possibly elements of general relativity (GR). Some knowledge of particle physics, group theory, and differential geometry would be helpful.

Why gauge theory of gravity? Because all the other fundamental interactions (electroweak and strong) are described successfully by gauge theories (of internal symmetries), whereas the established gravitational theory, Einstein’s GR, seems to be outside this general framework, even though, historically, the roots of gauge theory grew out of a careful analysis of GR. A full clarification of the gauge dynamics of gravity might be the last missing link to the hidden structure of a consistent unification of all the fundamental interactions at both the classical and the quantum level.

Our book is intended not just to be a simple reprint volume, but more a guide to the literature on gauge theories of gravity. The reader is expected first to study our introductory commentaries and become familiar with the basic ideas, then to read specific reprints, and after that to return again to our text, explore the additional literature, etc. The interaction is expected to be more complex than just starting with commentaries and ending with reprints. A student, guided by our commentaries, can get self-study insight into gauge theories of gravity within a relatively short period of time.

The underlying structure of gravitational gauge theory is the group of motions of the spacetime in SR, namely the Poincaré group $P(1,3)$. If one applies the gauge-theoretical ideas to $P(1,3)$, one arrives at the Poincaré gauge theory of gravity (PG). Therein, the conserved energy-momentum current of matter and the spin part of the conserved angular momentum current of matter both act as sources of gravity. The simplest PG is the...
Einstein–Cartan theory, a viable theory of gravity that, like GR, describes all classical experiments successfully. On the other hand, if one restricts attention to the translation subgroup of $P(1, 3)$, one ends up with the class of translation gauge theories of gravity, one of which, for spinless matter, can be shown to be equivalent to GR. The developments that led to PG are presented in Part A of our book; in Part B, definite and enduring results of PG are displayed. The content of Parts A and B should be considered as a mandatory piece of the general education for all gravitational physicists, while the remaining two parts cover subjects of a more specialized nature.

Since SR is such a well-established theory, from a theoretical as well as from an experimental point of view, the gauging of $P(1, 3)$ rests on a very solid basis. Nevertheless, there arise arguments as to why an extension of PG seems desirable; they are presented in Part C. As a finger exercise, we gauge the group of Poincaré plus scale transformations. Then, we extend $P(1, 3)$ to the general real linear group $GL(4, R)$, thus arriving at metric-affine gauge theory of gravity (MAG). This general framework leads to a full understanding of the role of a non-vanishing gradient of the metric (nonmetricity). Several other extensions treated in Part C appear to be rather straightforward tasks.

The gauge theory of gravity, since 1961, when it first had been definitely established, has had a broad development. Therefore, in Part D we display the results on several specific aspects of the theory, like the Hamiltonian structure, equations of motion for matter, cosmological models, exact solutions, three-dimensional gravity with torsion, etc. These subjects could be starting points for research projects for our prospective readers.

Clearly, making a good choice of reprints is a very demanding task, particularly if we want to take care of the historical justice and authenticity. But we also wanted to take care of another aspect—that our collection of reprints should be a useful guide to research-oriented readers without too many historical detours. These two aspects are not always compatible, and we tried to ensure a reasonable balance between them. To what extent these attempts were successful is to be judged by our readers.

- Chapters of the book that can be skipped at a first reading are marked by the star symbol *.
Acknowledgments

We are very grateful to the people who looked over early versions of our manuscript, helping us with detailed comments to improve the final form of the text: Peter Baekler (Düsseldorf), Giovanni Bellettini (Rome), Yakov Itin (Jerusalem), David Kerlick (Seattle), Claus Kiefer (Cologne), Bahram Mashhoon (Columbia, MO), Eckehard Mielke (Mexico City), Milan Mijić (Los Angeles), James M. Nester (Chungli), Yuri N. Obukhov (Moscow & Cologne), Hans Ohanian (Burlington), Dirk Puetzfeld (Bremen), Lewis Ryder (Canterbury), Tilman Sauer (Pasadena), Erhard Scholz (Wuppertal), Thomas Schücker (Marseille), Djordje Šijački (Belgrade), Andrzej Trautman (Warsaw) and Milovan Vasilić (Belgrade). The frontispiece on the classification of gauge theories was jointly created with Yuri Obukhov. One of us (MB) was supported by two short-term grants from the German Academic Exchange Service (DAAD), and the other one (FWH) is most grateful to Maja Burić (Belgrade) for an invitation to a workshop that took place in Divčibare, Serbia. FWH was partially supported by the German– Israeli Foundation for Scientific Research and Development (GIF), Research Grant No. 1078–107.14/2009. We also thank Ms. Hochscheid, Ms. Wetzels (both of Cologne), and Ms. Mihajlović (Belgrade) for technical support.

We wish to express our sincere gratitude to the publishing companies and the individuals who kindly granted us permissions to reproduce the material for which they hold copyrights: Acta Physica Polonica B, American Institute of Physics, American Physical Society, Peter Baekler, Caltech, Dover Publications, Elsevier, French Academy of Sciences, Indian Academy of Sciences, Institute of Physics, David Kerlick, Tom W. B. Kibble, Gertrud Kröner, Eric A. Lord, Dvora Ne’eman, James M. Nester, Wei-Tou Ni, Progress of Theoretical Physics, Dirk Puetzfeld, Royal Society of London, Ken Sakurai, Lidia D. Sciama, Società Italiana di Fisica, Springer Science+Business Media, Kellogg S. Stelle, William R. Stoeger, Paul K. Townsend, Andrzej Trautman, Paul von der Heyde, World Scientific, Chen Ning Yang and Hwei-Jang Yo.

We thank Professor Kibble, one of the founders of the gauge theory of gravity, who honored us by writing a foreword to this book.
List of useful books

Here is a chronologically ordered list of books, in which readers can find useful information on the subject of gauge theories of gravity. The selection is made by requiring at least some mentioning of the EC theory.

- W. Kopczyński and A. Trautman, Spacetime and Gravitation (PWN, Warsaw; Wiley, Chichester, 1992)
- M. Blagojević, Gravitation and Gauge Symmetries (IoP, Bristol, 2002)
Gauge-theoretic formalism (universal principle of the local invariance and the mechanism of spontaneous breaking of the gauge symmetry) forms the basis for the modern understanding of fundamental physical interactions and is successfully confirmed by the experimental discoveries of the gauge bosons and the Higgs particle. The carefully selected material of the book provides a minimal but sufficient mathematical introduction to the methods of the gauge gravitational theory, and gives a concise but exhaustive description of all specific physical consequences. In order to describe the Gauge Theories of Gravitation Blagojevic Milutin World Scientific Publishing 9781848167261: In the last five decades, the gauge approach to gravity has represented a research area of increased. First published in 1973, Gravitation is a landmark graduate-level textbook that presents Einstein's general theory of relativity and offers a rigorous, full-year course on the physics of gravitation. Upon publication, Science called it "a pedagogic masterpiece," and it has since become a classic, considered essential reading for every serious student and researcher in the field of relativity. This authoritative text has shaped the research of generations of physicists and astronomers, and the book continues to influence the way experts think about the subject. Einstein's theory of General Relativity that explains both the force of gravity and the shape of our universe has a mountain of evidence in its favor. And yet the way it was first developed had... Just a correction: Utiyama created gauge gravitation theory in 1956, not 1950. Gauge theories were created only in 1954. R Utiyama, "Invariant theoretical interpretation of interaction", Physical Review 101 (1956) 1597. Timo O. Korhonen. 8 months ago. It is true that The Gauge Theory gives some new perspectives to Einstein ideas. However, I think the quantum theory is most elementary to open up things still more. There, for instance a minimum length of space can be depicted, that is the Planck... Gravitation, gauge theories and differential geometry. Tohru EGUCHI Stanford Linear Accelerator Center, Stanford, California 94305, USA and The Enrico Fermi Institute and. Department of Physics, The University of Chicago, Chicago, Illinois, USA Peter B. GILKEY. Fine Hall, Box 37. Department of Mathematics, Princeton University, Princeton, New Jersey 08544, USA and Department of Mathematics, University of Southern California, Los Angeles, California 90007, USA. and Andrew J. HANSON Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720, USA and P.O. Box 11693A, Palo